Department of Energy

Bonneville Power Administration
P.O. Box 3621
Portland, Oregon 97208-3621

FREEDOM OF INFORMATION ACT/PRIVACY PROGRAM

April 8, 2016

In reply refer to: FOIA #BPA-2016-00727-F

Andrew Viccaro Zorn
DiGioia, Gray & Associates, LLC
570 Beatty Road
Monroeville, PA 15146

Dear Mr. Zorn,

This communication is the Bonneville Power Administration's (BPA) acknowledgment and final response to your Freedom of Information Act (FOIA) request, received by BPA on March 31, 2016. That FOIA request has been assigned tracking number BPA-2016-00727-F for your reference and use.

Request

> "M. W. Tuominen, '3 phase circuit model for ADSS optical fiber contamination currents,' Bonneville Power Admin., US Dept. of Energy, Engineering Report TNL3-99-1, May 1999."

Response

The record responsive to the above request, provided by BPA's Transmission Line Design personnel, accompanies this communication. With the release of the responsive record, comprising nine pages with no redactions, your FOIA request is now closed.

Fee

There are no FOIA fees applicable to the fulfillment of your request for BPA records.

Appeal

Pursuant to Department of Energy FOIA regulations at 10 C.F.R. § 1004.8, you may administratively appeal this response in writing, within 30 calendar days. If you choose to appeal, please include the following information with your written appeal:

1. The nature of your appeal- denial of records, partial denial of records, adequacy of search, or denial of fee waiver;
2. Any legal authorities relied upon to support the appeal; and
3. A copy of the determination letter.

Clearly mark both your appeal letter and envelope with the words "FOIA Appeal," and direct them to the following address:

Director, Office of Hearings and Appeals
Department of Energy
1000 Independence Avenue SW
Washington DC 20585-1615

If you have questions about this communication, please contact James King (CorSource Technology Group, Inc.), assigned to the BPA FOIA office, at 503-230-7621.

Sincerely,

C. M. Frost
Freedom of Information/Privacy Act Officer

Responsive record

3 Phase Circuit Model for ADSS Optical Fiber Contamination Currents
[BPA Report No. TNL3-99-1]

MONTY W. TUOMINEN, P.E.
Electrical Engineer

US Department of Energy
Bonneville Power Administration
Portland, Oregon - May, 1999
Word2000 - March 2004

INTRODUCTION

ADSS (all dielectric self supporting) optical fiber on high voltage structures is subject to electric fields of sufficient strength to cause corona, microsparking, and dry band arcing. Corona, usually on the supporting hardware, is eliminated with grading rings and microsparking is reduced with grading bars attached to the tower; however, at this date, dry band arcing, thought to be the leading cause of ADSS failures, is still under study in the United States and Great Britain.

By itself ADSS is non-conductive; however, accumulated contamination on the outer jacket can be very conductive. Capacitively induced currents in the contamination layer from adjacent energized conductors may lead to dry band arcing.

In this report a modeling technique is described which predicts voltages and currents in the conductive contamination layer along the ADSS span. This technique includes tower geometry, span length, sag of conductors and fiber, and contamination resistance. Appendix A covers the development of the system equations and describes an easily implemented "backward-forward" solution algorithm. Appendix B explains calculation of capacitance between conductors and appendix C shows the conductor sag algorithm. An Excel spreadsheet has been written which uses these methods.

For illustrative purposes, the technique is applied to a BPA 500 kV delta tower (238 series) and the outcome is surprising.

ARC GENERATION and CONTAMINATION

Dry band arcing is believed to be caused by milliampere sized currents in the contamination layer on the ADSS jacket. When wet contamination begins to dry, voltage across a dry band may be sufficient to initiate an arc and, if the available current is high enough, the arc heating can degrade the underlying ADSS jacket [1].

ADSS jacket types are commonly divided into two categories: "standard" and "track resistant". Preliminary studies at BPA have indicated that arcs of about one milliampere will damage standard jackets but are resisted almost indefinitely by track resistant jackets[4]. The same tests showed that currents must approach 5 milliamperes (or more) to initiate significant damage in track resistant jackets [Note: The applied voltage was 26 kV in these tests. Further work at BPA will hopefully classify combinations of arc voltages and currents leading to jacket damage].

Contamination can be both natural and man-made. Salt fog as is found is the British Isles is perhaps the most conductive and reported to be the main source of ADSS damage in that country [1,2]. Dust from alkali lake beds in Eastern Oregon is another natural contaminant though uncertain in magnitude compared to salt. Man-made sources include agricultural fertilizers and pesticides, vehicle exhaust, and factory emissions in general (cement plants are most notable).

It is assumed in this report that contamination can be quantified by ohmic resistance in units per length such as ohms per meter. Research reports from Britain describe contamination in three basic levels: heavy, medium, and light corresponding to 10^5, 10^6, and 10^7 ohm/meter respectively [2]. The 10^5 and 10^6 ohm/meter levels are demonstrated in the example in this report.

THE LUMPED CIRCUIT MODEL

Transmission line electrical characteristics are often described by distributed inductance, capacitance, and resistance per length. Inductance can be ignored for ADSS (span lengths are very short compared to 60 Hz wavelengths) which permits the model as described herein. Figure 1 is a general picture of a 500 kV line with ADSS. Figure 2 depicts distributed parameters of the lumped circuit ADSS model. Figure 3 is the exact tower geometry used in the example.

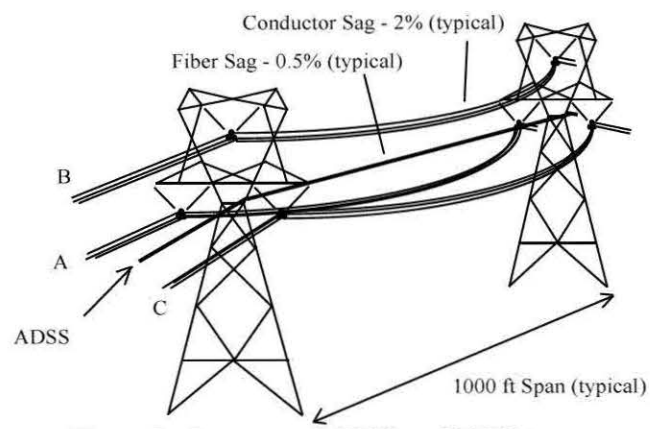


Figure 1. One span of ADSS on 500 kV towers with typical sags noted.

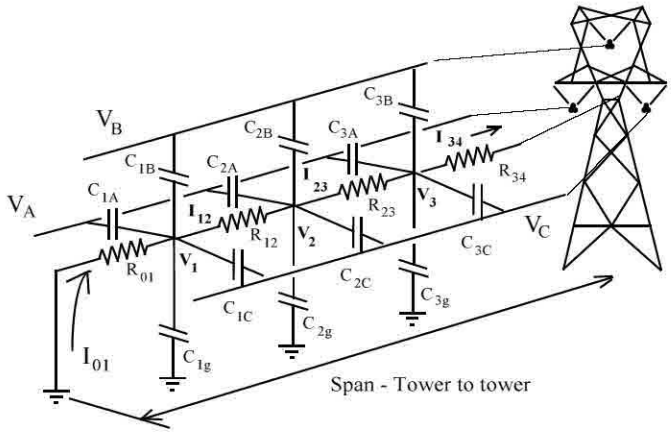


Figure 2. Distributed parameters of the lumped circuit ADSS model (also see figure A1 in appendix A)

In figure 2, each section consists of the total contamination resistance within the section (R_{01}, R_{12}, R_{23}, etc.), the capacitance between each phase and the ADSS contamination layer (C_{1A}, C_{1B}, C_{1C}, C_{2A}, etc.), and the capacitance between the layer and ground (C_{1g}, C_{2g}, etc.). V_A, V_B, and V_C are the conductor voltages. V_1, V_2, etc. and I_{01}, I_{12}, I_{23}, etc. are the induced voltages and currents on the ADSS contamination.

Details of the solution and implementation algorithm for the voltages and currents are in appendix A, the method for calculation of line capacitances is in appendix B, and the conductor sag algorithm is in appendix C. An Excel spreadsheet has been developed using the algorithms and is the source of the charts in this report.

In most situations I_{01} and $I_{n,n+1}$ are the largest currents and enter the grounded hardware at the towers (location of most reported failures). But analysis will reveal that the largest current could be out on the span under certain conditions.

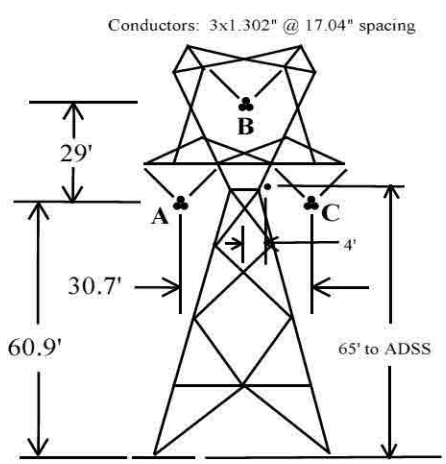


Figure 3. Dimensions of 500 kV 238 Series tower with ADSS - used in the example

UNPERTURBED SPACE POTENTIAL
Brief comments

Unperturbed space potential is a criteria used many designers to locate ADSS (usually at 25 kV or less for track resistant jackets) on a structure. "Unperturbed" means that the electrostatic influence of the tower is ignored. Conductors are considered infinitely long - with no sag - and the space potential can be found via a straightforward 2D electric field calculation.

Fortunately, electric field activity near the tower is relatively unaffected by the sagging conductor away from the tower. The 2D method can work very well for properly positioning ADSS. However, as contamination accumulates and small currents begin to flow then midspan geometry has an effect on overall induced currents and voltages. Sag does have an effect. Variation of "unperturbed" space potential as a function of sag is interesting but doesn't appear to be as useful as actual induced quantities.

500 kV TOWER STUDY

The following series of charts show the capabilities of the Excel spreadsheet as well as give an example of possible contamination situations. Figure 3 is the tower design selected for the study. A 1000 foot span was selected as typical, conductor sags were 2% and the ADSS was sagged at 0.5%. ADSS is located near an "unperturbed" 45 kV at the tower. Figure 4, below, shows sag profiles and space potential long the ADSS.

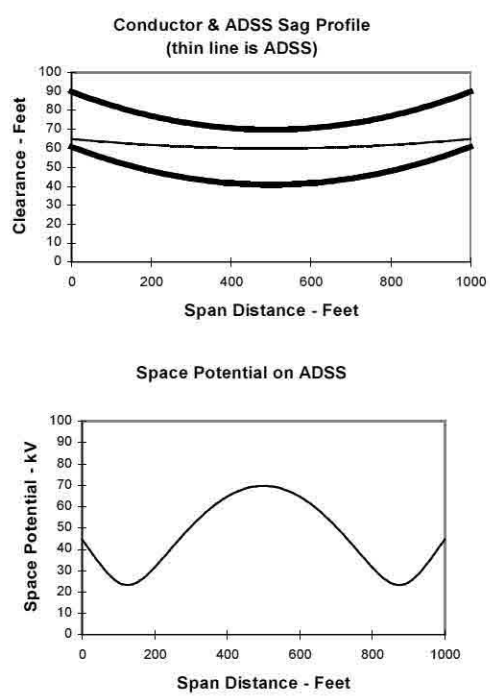


Figure 4. Sag profiles and ADSS Space Potentials Conductors at 2%, ADSS at 0.5%

Obviously, along the ADSS, variation in space potential is due to the difference in sag between the ADSS and the conductors. At midspan the space potential is about 70 kV.

If the ADSS were sagged to match the conductors the potential one would expect little variation which is shown in figure 5.

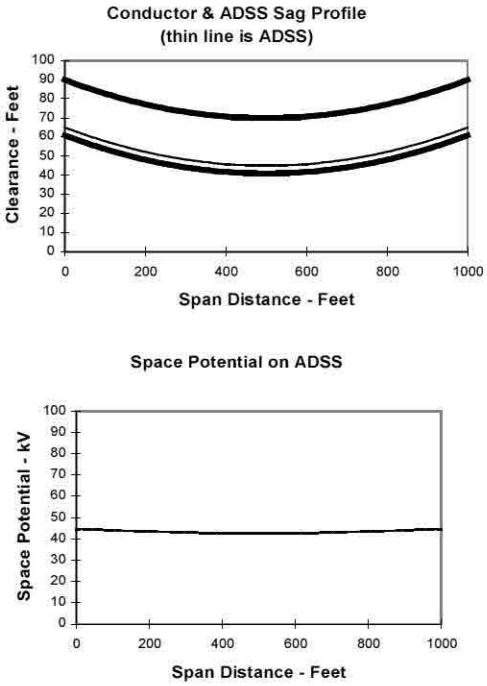


Figure 5. Sag profiles and ADSS Space Potentials Conductors at 2%, ADSS at 2%

Comparing figure 4 to figure 5 illustrates the effect of sag on the ADSS space potential. As previously mentioned, electric field activity near the tower (corona on hardware, microsparking) is little affected by the midspan situation. There might be a slight reduction because the space potential (figure 4) does initially decrease away from the tower - down to nearly 25 kV before going up to 70 kV.

What effect is predicted if contamination is sufficient to cause dry band arcing? By introducing resistance low enough to create milliampere sized currents one can observe that there is an effect from the variations in sag. And, as will be demonstrated, when ADSS is sagged differently from the main conductors, other factors may have an effect.

The next four charts were computed using 10^5 ohm/meter resistance, considered "heavy" in the British studies. Note that the induced current near the tower is about 8 milliamperes.

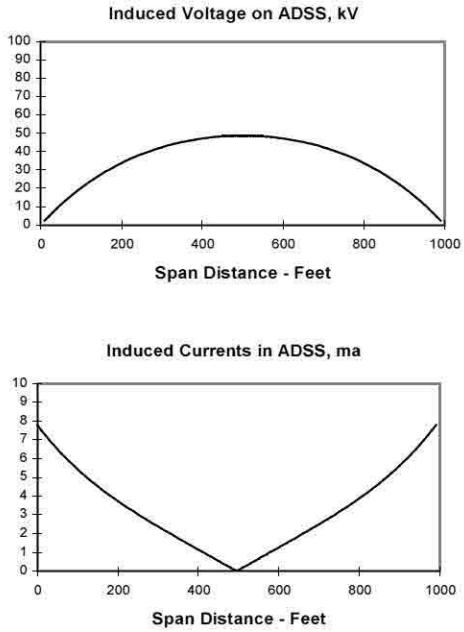


Figure 6. Induced voltage and current on ADSS with sag matching conductor - both at 2%

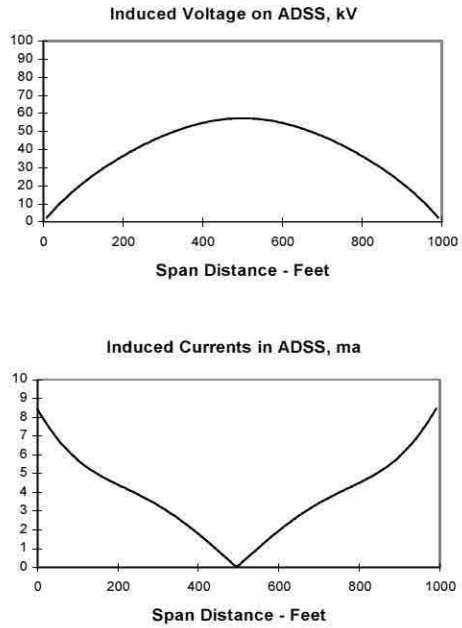


Figure 7. Induced voltage and current on ADSS with sag at 0.5%, conductor at 2%

By comparing figures 6 and 7 one concludes that sag difference has some but not earth shattering effect on the induced current distribution, at least for this example. Induced current near the tower increased from slightly below to slightly above 8 milliamps. However, the next set of figures reveal what happens when conductor phasing is reversed, that is , from ABC to ACB.

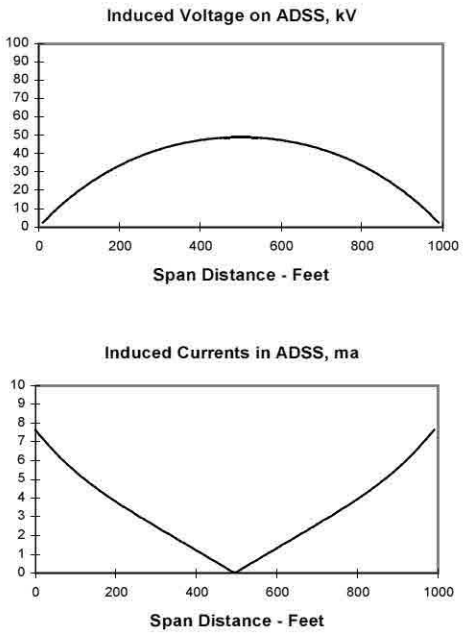


Figure 8. Induced voltage and current on ADSS with sag matching conductor - both at 2%. Phase reversal on main conductors.

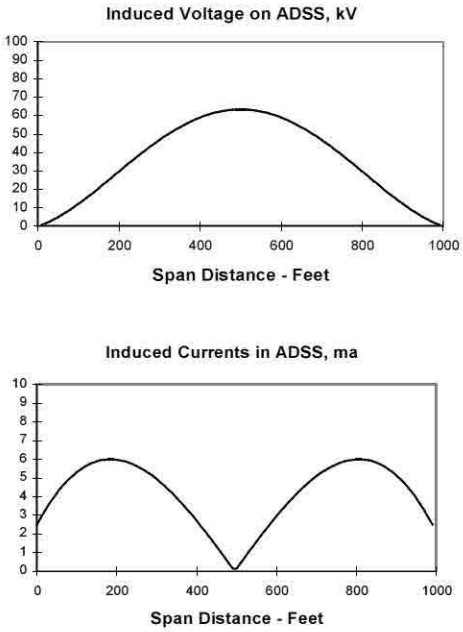


Figure 9. Induced voltage and current on ADSS with sag at 0.5%, conductor at 2% Phase reversal on main conductors.

Comparison of figures 7 and 9 shows the astounding effect of phase reversal. Induced current at the tower has been reduced from about 8 to nearly 2.5 milliamps. Figure 8, however, is hardly different from figure 6. The author found that not only phase reversal but also moving the ADSS to the opposite side of the structure produced the same results. What happens if contamination is less?

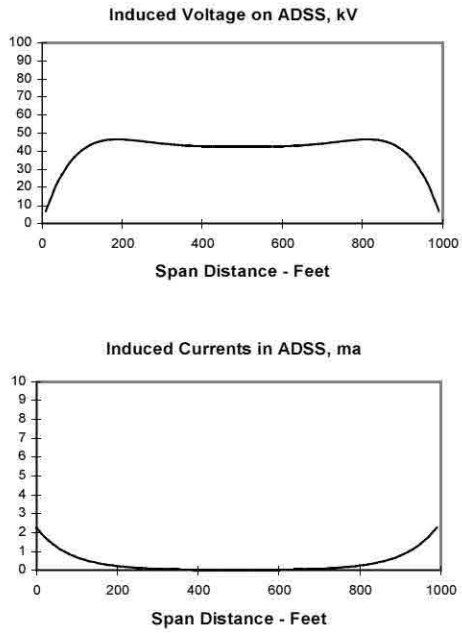


Figure 10. Same as figure 8 (opposite column) with resistance increased from 10^5 to 10^6 ohms/meter (reduced contamination)

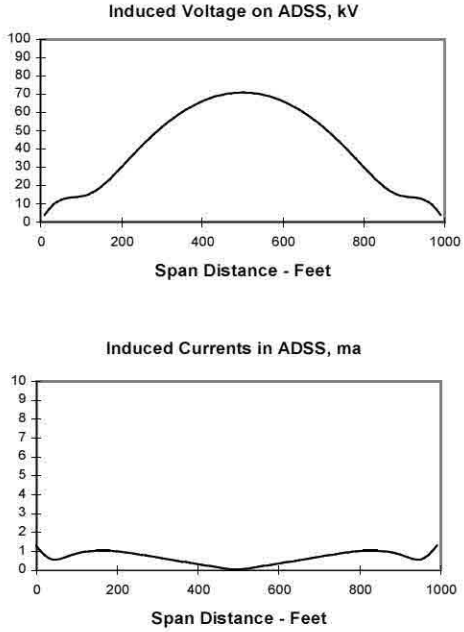


Figure 11. Same as figure 9 (opposite column) with resistance increased from 10^5 to 10^6 ohms/meter (reduced contamination)

Figures 8 and 9 translate to figures 10 and 11 and the contamination resistances are 10^5 and 10^6 ohms/meter respectively. Phase sequence has a pronounced effect on current and voltage distribution along the span. Note that the 2+ milliamps at the tower in figure 10 is reduced to nearly 1 milliamp in figure 11.

CONCLUSIONS

Phase sequence seems to have an effect which may be enhanced by differences in sag between ADSS and the conductors.

The implication of the study results, pending further investigations, is that, unless all sags and conductor phasing are included in the calculations, predictions of induced currents may be in error. For example, one may have predicted 2 or 3 milliamps and measured over 8 in a field situation - or vice versa - and concluded wrongly that there was a problem in instrumentation or method.

The Excel spreadsheet can accommodate up to 9 phases permitting analysis of double circuit towers including ground wires. Preliminary investigation indicates that sag and phasing definitely have effects on other tower designs but the amount of effect appears to depend on the specific design. The author hopes to provide another report comparing various situations.

References

[1] C. N. Carter and M. A. Waldron , "Mathematical model of dry band arcing on self-supporting all dielectric optical cables strung on overhead power lines', IEE Proceedings - C.Vol. 139, No. 3, p. 185-196, May 1992.

[2] C. N. Carter, J. Deas, N. R. Haigh, and S. M. Rowland, "Applicability of all-dielectric self supporting cable systems to very high voltage overhead power lines", 46th Proc. International Wire and Cable Symposium, p. 622-631, 1997.

[3] (Appendix) Transmission Line Reference Book, 345 kV and Above, Second Edition (a.k.a. "GE Redbook"), EPRI, 1982.

[4] M. D. Johnson and J. O. Lo, "ADSS Jacket Arc Resistant Material Tests", Laboratory Services Report #TNF(M)-98-24a, Bonneville Power Admin., US Dept. of Energy, January 21, 1999.

APPENDIX A
Solution for ADSS currents and voltage

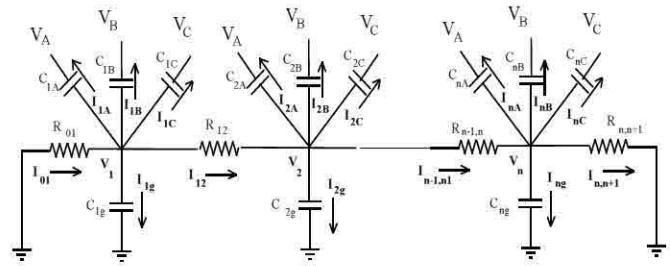


Figure A1. Distributed parameters of the lumped circuit ADSS model (also see figure 2, page 2)

The above schematic depicts the parameters of a span of ADSS divided into *n* nodes. Current direction can be arbitrary; according to the schematic the current summation for each node is:

$$I_{01} = I_{1A} + I_{1B} + I_{1C} + I_{1g} + I_{12}$$
$$I_{12} = I_{2A} + I_{2B} + I_{2C} + I_{2g} + I_{23}$$
$$\vdots$$
$$I_{n-1,n} = I_{nA} + I_{nB} + I_{nC} + I_{ng} + I_{n,n+1}$$

Replacing currents by voltage difference and impedances:

$$\frac{-V_1}{R_{01}} = \frac{V_1-V_A}{X_{1A}} + \frac{V_1-V_B}{X_{1B}} + \frac{V_1-V_C}{X_{1C}} + \frac{V_1}{X_{1g}} + \frac{V_1-V_2}{R_{12}}$$

$$\frac{V_1-V_2}{R_{12}} = \frac{V_2-V_A}{X_{2A}} + \frac{V_2-V_B}{X_{2B}} + \frac{V_2-V_C}{X_{2C}} + \frac{V_2}{X_{2g}} + \frac{V_2-V_3}{R_{23}}$$
$$\vdots$$
$$\frac{V_{n-1}-V_n}{R_{n-1,n}} = \frac{V_n-V_A}{X_{nA}} + \frac{V_n-V_B}{X_{nB}} + \frac{V_n-V_C}{X_{nC}} + \frac{V_n}{X_{ng}} + \frac{V_n}{R_{n,n+1}}$$

where $X_{nm} = (j\omega C_{nm})^{-1}$. Separation of the unknown voltages, V_1 through V_n , and algebraic manipulation yields the convenient matrix form:

$$\begin{bmatrix} G_1 & \frac{-1}{R_{12}} & 0 & 0 & \dots & 0 \\ \frac{-1}{R_{12}} & G_2 & \frac{-1}{R_{23}} & 0 & \dots & 0 \\ 0 & \frac{-1}{R_{23}} & G_3 & \frac{-1}{R_{34}} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \frac{-1}{R_{n-1,n}} & & G_n \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_n \end{bmatrix}$$

Where: $G_n = \frac{1}{R_{n-1,n}} + j\omega(C_{nA} + C_{nB} + C_{nC} + C_{ng}) + \frac{1}{R_{n,n+1}}$

and: $I_n = j\omega(V_A C_{nA} + V_B C_{nB} + V_C C_{nC})$

Theoretically, the matrix form will accommodate an infinite number of nodes. The author has implemented solutions in 100 and 500 node Excel spreadsheets as well as Fortran code up to 10,000 nodes. The 100 node Excel spreadsheet was, of course, the fastest even if slightly less accurate - within 3 % of a 10,000 node Fortran solution. A 300 mhz PC with Excel 97 can perform the 100 node computation in about five seconds (Note: Excel 4 Macro language was employed for two reasons: author familiarity and backward capability of Excel 5. Excel 97 will run the "old" macros does not provide the language).

Programmers can use any preferred matrix techniques for obtaining Vn. However, the large number of zeros and limited number of terms in each equation prompted the author to devise a simple recursive substitution easily set up in Fortran (example bottom of next column), Excel 4 macro, C, or any math oriented language. Complex number matrix routines are avoided (not available in Excel) and execution time shortened.

The following derivation is somewhat laborious (reason why it is in appendix; also, apologies to those mathematically inclined minds who find this trivial) but is intended to show how the method was obtained. In the following sample 4x4 matrix, A_n are the G_n terms, B_n are the off diagonal resistance terms and D_n are the source or I_n terms (the author found ABD notation easier to follow). And, x_n are the classic unknowns.

$$\begin{bmatrix} A_1 & B_1 & 0 & 0 \\ B_1 & A_2 & B_2 & 0 \\ 0 & B_2 & A_3 & B_3 \\ 0 & 0 & B_3 & A_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

Begin by "backward zeroing out" all of the upper B_n . The first step of this process results in:

$$\begin{bmatrix} A_1 & B_1 & 0 & 0 \\ B_1 & A_2 & B_2 & 0 \\ 0 & B_2 & A_3' & 0 \\ 0 & 0 & B_3 & A_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3' \\ D_4 \end{bmatrix}$$

where by manipulating the equations it is noted that

$$A_3' = A_3 - \frac{B_3^2}{A_4} \quad \text{and} \quad D_3' = D_3 - \frac{B_3 D_4}{A_4}$$

Then:

$$\begin{bmatrix} A_1 & B_1 & 0 & 0 \\ B_1 & A_2' & 0_2 & 0 \\ 0 & B_2 & A_3' & 0 \\ 0 & 0 & B_3 & A_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2' \\ D_3' \\ D_4 \end{bmatrix}$$

where:

$$A_2' = A_2 - \frac{B_2^2}{A_3'} \quad \text{and} \quad D_2' = D_2 - \frac{B_2 D_3'}{A_3'}$$

Finally:

$$\begin{bmatrix} A_1' & 0 & 0 & 0 \\ B_1 & A_2' & 0_2 & 0 \\ 0 & B_2 & A_3' & 0 \\ 0 & 0 & B_3 & A_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} D_1' \\ D_2' \\ D_3' \\ D_4 \end{bmatrix}$$

where:

$$A_1' = A_1 - \frac{B_1^2}{A_2'} \quad \text{and} \quad D_1' = D_1 - \frac{B_1 D_2'}{A_2'}$$

The solution for x_n is simply by the forward order:

$$x_1 = \frac{D_1'}{A_1'}$$

$$x_2 = \frac{D_2' - x_1 B_1}{A_2'}$$

$$x_3 = \frac{D_3' - x_2 B_2}{A_3'}$$

$$x_4 = \frac{D_4 - x_3 B_3}{A_4}$$

In Fortran notation the above steps can be summarized for N nodes:

```
DO  K =  N-1,1,-1          (descending order)
  A(K) = A(K)  -  B(K)*B(K)/A(K+1)
  D(K) = D(K)  -  B(K)*D(K+1)/A(K+1)
ENDDO
X(1) = D(1)/A (1)          (1st term)
DO   K = 2,N               (ascending order)
  X(K) =  [D(K)  - X(K-1)*B(K-1)]/ A(K)
ENDDO
```

where A, B, D and X are complex arrays. Note that A and D are replaced with the equivalent of A' and D'.

Relating the ABD terms back to electrical parameters:

Where: $A_n = \frac{1}{R_{n-1,n}} + j\omega(C_{nA} + C_{nB} + C_{nC} + C_{ng}) + \frac{1}{R_{n,n+1}}$

$$B_n = \frac{-1}{R_{n,n+1}}$$

and $D_n = j\omega(V_A C_{nA} + V_B C_{nB} + V_C C_{nC})$

APPENDIX B
Calculation of transmission line capacitances

This section is intended for those readers less familiar with the commonly employed equations for calculating coupling capacitances between conductors and between conductors and ground.

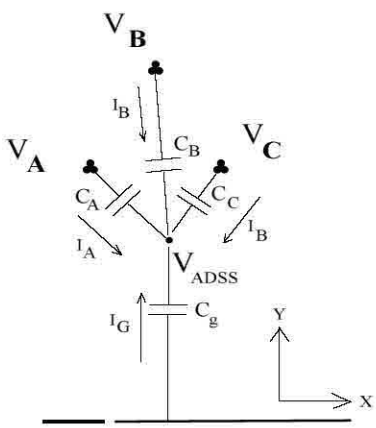
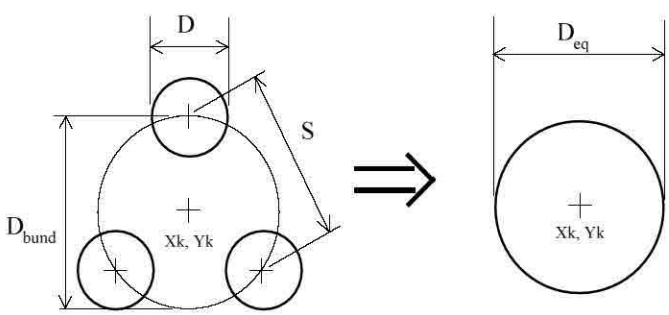


Figure B1. ADSS coupling capacitances with 3 phases

First, bundled "subconductors" are reduced to a single conductor of an electrically equivalent diameter as shown in the following diagram.



where

D	=	Diameter of single subconductor
S	=	Subconductor adjacent spacing
D_{bund}	=	Bundle diameter (calculated)
D_{eq}	=	Diameter of electrical equivalent conductor
n	=	number of conductors in bundle (3 in example)
Xk, Yk	=	location of kth bundle in Cartesian coordinates (center of tower usually at X = 0)

If m = 1 then $D_{eq} = D$.

If m is 2 or more then

$$D_{bund} = \frac{S}{\sin(180/m)}$$

and

$$D_{eq} = D_{bund}(mD/D_{bund})^{1/m}$$

Some common equivalencies at BPA are (units given in inches)

m	x	D	@	S	= 1 x D_{eq} (formula)
3	x	1.302	@	17.04	= 1 x 11.478
2	x	1.602	@	18.00	= 1 x 7.594
3	x	1.602	@	19.75	= 1 x 13.571

Next, the inter-conductor capacitances are derived from Maxwell's potential coefficients. The general theory states that voltage on a conductor is the sum of voltages induced by its own line charge and line charges on all other conductors [3]. The equations are:

$$\begin{aligned}
V_1 &= P_{11}Q_1 + P_{12}Q_2 + P_{13}Q_3 + \ldots P_{1k}Q_k \\
V_2 &= P_{21}Q_1 + P_{22}Q_2 + P_{23}Q_3 + \ldots P_{2k}Q_k \\
V_3 &= P_{31}Q_1 + P_{32}Q_2 + P_{33}Q_3 + \ldots P_{3k}Q_k \\
&\vdots \\
V_k &= P_{k1}Q_1 + P_{k2}Q_2 + P_{k3}Q_3 + \ldots P_{kk}Q_k
\end{aligned}$$

where the P_{kk} are Maxwell coefficients relating voltage to line charge. In matrix form:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_k \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \ldots P_{1,k} \\ P_{21} & P_{22} & P_{23} & \ldots P_{2,k} \\ P_{31} & P_{32} & P_{33} & \ldots P_{3,k} \\ \vdots & \vdots & \vdots & \vdots \\ P_{k,1} & P_{k,2} & P_{k,3} & \ldots P_{kk} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ \vdots \\ Q_k \end{bmatrix}$$

where

$$P_{kk} = 1.8\text{x}10^{10} \ln(4\,Y_k / D_{k,eq}))$$

and

$$P_{jk} = 1.8\text{x}10^{10} \ln\left[\frac{[(Xj - Xk)^2 + (Yj + Yk)^2]^{1/2}}{[(Xj - Xk)^2 + (Yj - Yk)^2]^{1/2}}\right] \quad j \neq k$$

P_{kk} is the self induced voltage coefficient and P_{jk} is the influence of the kth conductor charge Q_k on jth conductor voltage V_j. Capacitances can be shown via short matrix form:

$$\begin{aligned}
[V] &= [P][Q] \\
[P]^{-1}[V] &= [P]^{-1}[P][Q] \\
[C][V] &= [Q]
\end{aligned}$$

where $[C] = [P]^{-1}$

Matrix inversion of "P" yields "C" and the system now has the form:

$$\begin{bmatrix} C_{11} & -C_{12} & -C_{13} & ..-C_{1,k} \\ -C_{21} & C_{22} & -C_{23} & ..-C_{2,k} \\ -C_{31} & -C_{32} & C_{33} & ..-C_{3,k} \\ \vdots & \vdots & \vdots & \vdots \\ -C_{k,1} & -C_{k,2} & -C_{k,3} & ...C_{kk} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_k \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ \vdots \\ Q_k \end{bmatrix}$$

All of the diagonal terms will be positive, the rest will be negative, and by symmetry $C_{12} = C_{21}$, $C_{13} = C_{31}$, etc.

How do the C_{kk} capacitances above relate to C_A, C_B, C_A, and C_g in figure B1? Furthermore, can the negativity of the off-diagonal be verified?

For discussion let k = 4 which corresponds to figure B1 where the ADSS is the 4th "conductor" (note: ADSS is included because, although by itself an insulator, the accumulation of contamination causes it to electrically respond like a high resistance conductor). The capacitance matrix is:

$$\begin{bmatrix} C_{11} & -C_{12} & -C_{13} & -C_{14} \\ -C_{12} & C_{22} & -C_{23} & -C_{24} \\ -C_{13} & -C_{23} & C_{33} & -C_{34} \\ -C_{14} & -C_{24} & -C_{34} & C_{44} \end{bmatrix}$$

Assuming phase A is conductor 1, phase B is conductor 2, phase C is conductor 3, and the ADSS is conductor 4, in figure B1 it can be shown that:

$$C_A = C_{14}$$
$$C_B = C_{24}$$
$$C_A = C_{34}$$
$$C_g = C_{44} + (-C_{14} - C_{24} - C_{34})$$

To show this we know that by Kirchoff's Current Law the sum of currents entering the ADSS node is zero:

$$0 = I_A + I_B + I_C + I_G + I_f + I_b$$

where I_f and I_b are currents into the front and back of the node along the ADSS. By substituting the voltage differences and capacitive coupling reactance:

$$0 = (V_1 - V_4)j\omega C_A + (V_2 - V_4)j\omega C_B + (V_3 - V_4)j\omega C_C + (- V_4)j\omega C_g + I_f + I_b$$

and by rearranging terms:

$$I_f + I_b = j\omega[-V_1C_A - V_2 C_B - V_3C_C + V_4(C_A + C_B + C_C + C_g)] \qquad \textbf{Equation 1}$$

Now note the charge equation from the capacitance matrix developed by Maxwell coefficients (opposite column):

$$-V_1C_{14} - V_2 C_{24} - V_3C_{34} + V_4C_{44} = Q_4$$

and since current is the time derivative of charge:

$$I = \frac{dQ}{dt} = C\frac{dV}{dt} = C\,(j\omega V)$$

then

$$j\omega(-V_1C_{14} - V_2 C_{24} - V_3C_{34} + V_4C_{44}) = \frac{dQ_4}{dt} = I_4$$

Equation 2

Since Q_4 is line charge then I_4 is line displacement current and equal to the sum of I_f and I_b from equation 1. Comparing **equations 1** and **2** it can be seen that:

$$C_{14} = C_A$$
$$C_{24} = C_B$$
$$C_{34} = C_C$$
$$C_{44} = C_A + C_B + C_C + C_g$$

Therefore: $C_g = C_{44} - C_{14} - C_{24} - C_{34}$ (QED!)

Unperturbed space potential can also be found from these equations. If it is assumed there is no charge or current then:

$$V_4 = \frac{(V_1C_{14} + V_2 C_{24} + V_3C_{34})}{C_{44}}$$

and it follows that, in reference to figure B1:

$$V_{ADSS} = \frac{(V_AC_A + V_B C_B + V_CC_C)}{C_A + C_B + C_C + C_g}$$

Laboratory tests often use single phase sources. In this case:

$$V_{ADSS} = \frac{V_AC_A}{C_A + C_g} \quad \text{or} \quad \frac{V_AC_{14}}{C_{44}}$$

By substituting the capacitive reactance, $X = 1/j\omega C$, we find that:

$$V_{ADSS} = V_A \cdot \frac{X_g}{X_g + X_A}$$

which is the well known simple voltage divider.

A final note: The coefficient 1.8×10^{10} is derived from e_0, the permittivity of space, which has the units farads/meter. Therefore, all dimensions must be converted to meters for compatibility with the calculations. Resulting capacitance units are farads/meter. These values as well as resistance in ohms/meter must be adjusted according the distance between nodes in order to obtain the proper lumped values of R and C (figure 2 and figure A1).

APPENDIX C
The sag algorithm

This section describes the algorithm used to establish variation in clearance along a flat span based on percentage of sag. Unit weight or tension is not required; the algorithm guesses tensions and then iterates until a tension is found which provides the desired sag.

The basic catenary equation is:

$$Y_{sag} = \frac{H}{w} \left[\cosh \left(X \frac{w}{H} \right) - 1 \right]$$

where Y is the vertical displacement, X is the horizontal displacement from the center of the span, H horizontal tension, and w is the weight per unit length.

First, the desired maximum Y_{sag} is found from the span length and percent sag:

$$Y_{sag} = \% \text{ sag} \times \text{Span}$$

H is given an initial value of 1000 and w is assumed to be one. The following example shows beginning and final steps of the trial and error algorithm.

The example is for the 1000 foot span sagged at 2% used in the main section of this report.

1) $Y_{SAG} = \% \text{ sag} \times \text{Span} = .02 \times 1000 = 20$

$H_{GUESS} = 1000$ and $w = 1$.

$X = \text{Span} / 2 = 500$

2) $Y_{GUESS} = H_{GUESS} \left[\cosh (500 / H_{GUESS}) - 1 \right]$

3) $H_{NEW} = H_{GUESS} * Y_{GUESS} / Y_{SAG}$

4) $H_{GUESS} = H_{NEW}$

5) Perform 25 iterations of steps 2) through 4)

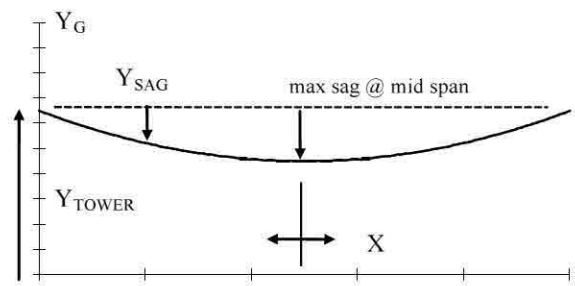
6) $H_{FINAL} = H_{GUESS} = 6253.330493$

7) If $(Y_{GUESS} - Y_{SAG}) / Y_{SAG}$ is greater than .01 then assume a convergence problem.

However, in the author's experience, 25 iterations is more than adequate. Only four iterations were required to Converge to the proper sag and H_{FINAL} in the example.

Thus, clearances can be established anywhere along the span. For the example the sag equation would be:

$$Y_{SAG}(X) = 6253.33... [\cosh (X / 6253.33...) - 1]$$

and the clearance to ground is

$$Y_G(X) = Y_{TOWER} - Y_{SAG}(X)$$